

LoopFest 2002 – BNL

**NLO top-pair production
at e+e- colliders**

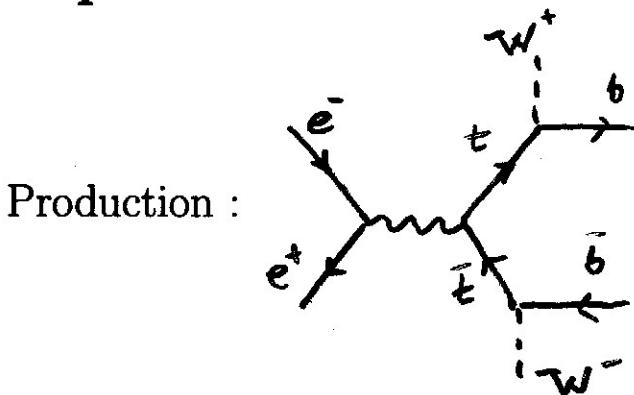
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Oklahoma State University

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Rochester University

Top studies at a Linear Collider



- Physics at the threshold ($W = 350 \text{ GeV}$)

Comprehensive theoretical studies : NNLO computation with resummation of large logarithms, elimination of the renormalon ambiguity...
(Melnikov & Yakovlev, Teubner & Hoang ...)

Results : – m_t to better than 200 MeV precision
– α_s , Γ_t and top Yukawa coupling to several percent

- Physics above threshold

Can study – top quark couplings
– spin correlations
– look for CP violation ...

At $W = 500 \text{ GeV}$ CM energy
 500 pb^{-1} integrated luminosity $\Leftrightarrow 3 \cdot 10^5$ top quark pairs created
 \Rightarrow experimental accuracy to better than 1%

Need similar precision in theoretical predictions !

QCD Radiative Corrections

- production : - virtual and soft gluon

Jersak, Laerman & Zerwas ('82)

- hard gluon

Korner *et al.*, Parke *et al.*,

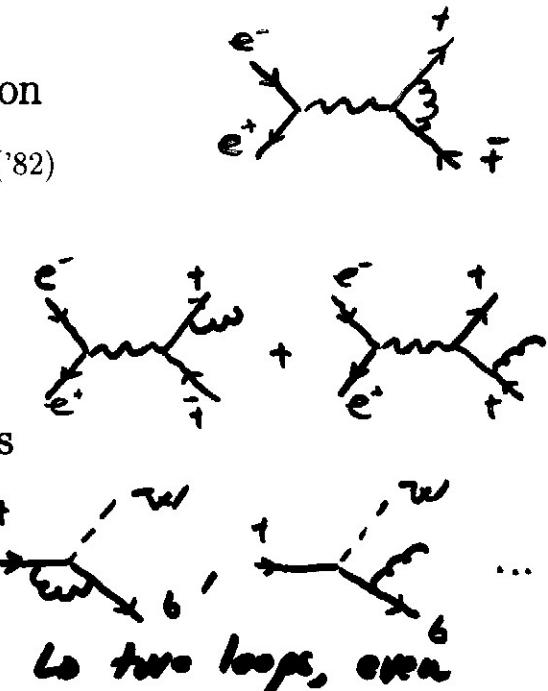
Brandenburg *et al.*, ...

- decay : virtual + hard gluons

Jezabek & Kuhn,

Lee, Oakes & Yuan, ...

A. Czarnecki



→ Top on-shell (narrow width approximation) C. Schmidt

$$\hookrightarrow e^+e^- \rightarrow t\bar{t}(g) \rightarrow 6W^+ 6\bar{W}^- (g)$$

Effects due to top being off-shell :

interference between subprocesses

magnitude : naively, of order Γ_t/m_t , but possibly larger
in differential cross-sections. $\sim 1\%$

- soft gluon radiation Khoze, Orr & Stirling ('94)

→ Top off-shell - soft + hard gluon **Macesanu & Orr**
- virtual corrections *** Macesanu**

2 Feynman diagrams – resonant behavior

- Physical process : $e^+e^- \rightarrow b W^+ \bar{b} W^-$

TREE LEVEL

- a) doubly resonant diagram :

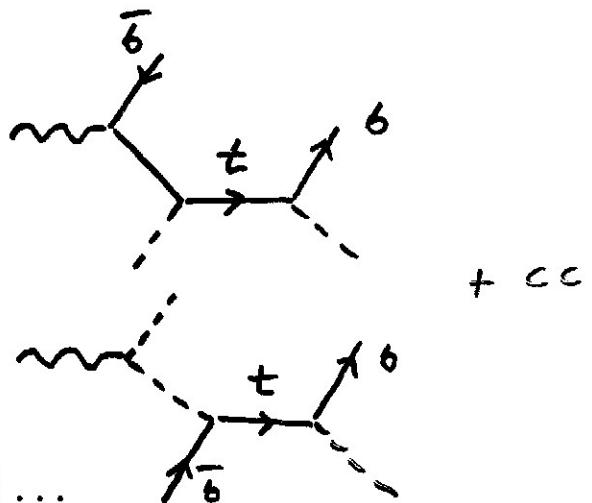
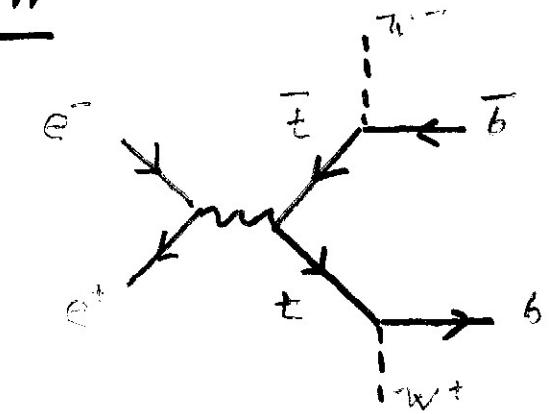
$$M_0 \sim \frac{1}{p_t^2 - \bar{m}_t^2} \frac{1}{p_t^2 - \bar{m}_t^2}$$

$$\bar{m}_t^2 = m_t^2 - im_t\Gamma_t$$

- b) singly resonant diagrams :

$$M_0 \sim \frac{1}{p_t^2 - \bar{m}_t^2}$$

- c) nonresonant diagrams : about 50 ...



- computation done in double pole approximation (DPA):

– keep only the amplitudes which have a doubly resonant behavior when $p_\perp^2 \approx p_{\bar{\tau}}^2 \approx m_\tau^2$

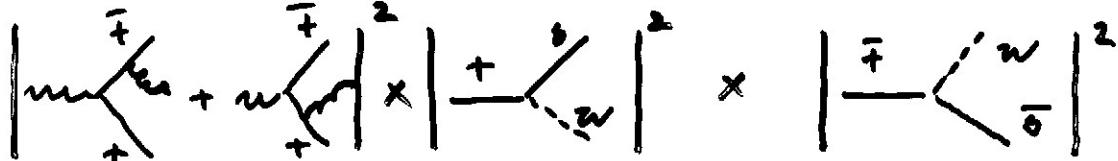
- Similar approach has been used in evaluating the cross section for W pair production at LEP.¹

(talks by Jegerlehner, Hollik, Dittmaier...)

Real Gluon Radiation

- on-shell

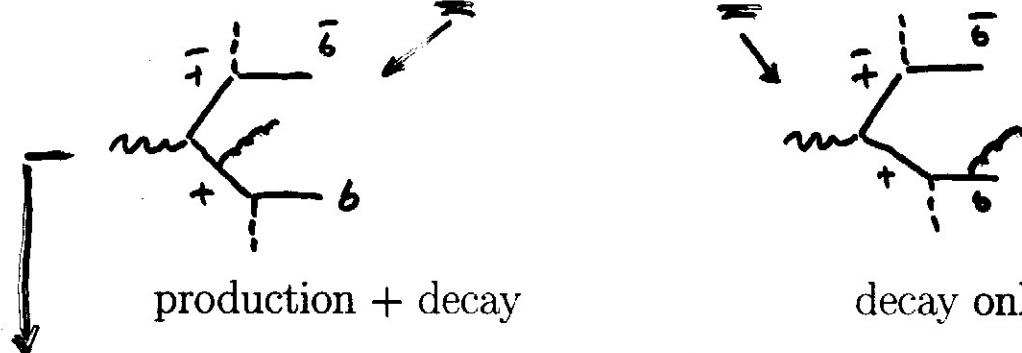
$$d\sigma_{prod}^{os} = d\sigma(ee \rightarrow ttg) \times d\sigma(t \rightarrow bW^+)/\Gamma_t \times d\sigma(\bar{t} \rightarrow \bar{b}W^-)/\Gamma_{\bar{t}}$$



similar expressions for $d\sigma_{tdec}^{os}$, $d\sigma_{\bar{t}dec}^{os}$

- off-shell

$$M = M_t + M_{\bar{t}} + M_b + M_{\bar{b}}$$



$$\frac{1}{(p+k)^2 - \bar{m}^2} \frac{1}{p^2 - \bar{m}^2} \rightarrow \frac{1}{2p \cdot k} \left[\frac{1}{p^2 - \bar{m}^2} - \frac{1}{(p+k)^2 - \bar{m}^2} \right]$$

$$P = P_w + P_b, \quad \bar{m}^2 = m_t^2 - im_t \Gamma_t \quad (\text{Khoze, Orr \& Stirling })$$

$$\underline{M = M_{prod} + M_{tdec} + M_{\bar{t}dec}}$$

$$\sigma \sim |M|^2 \sim |M_{prod}|^2 + |M_{tdec}|^2 + |M_{\bar{t}dec}|^2 + \sigma_{interf}$$

$$\sigma_{interf} = 2\text{Re} [M_{prod}(M_{tdec} + M_{\bar{t}dec})^* + M_{tdec} M_{\bar{t}dec}^*]$$

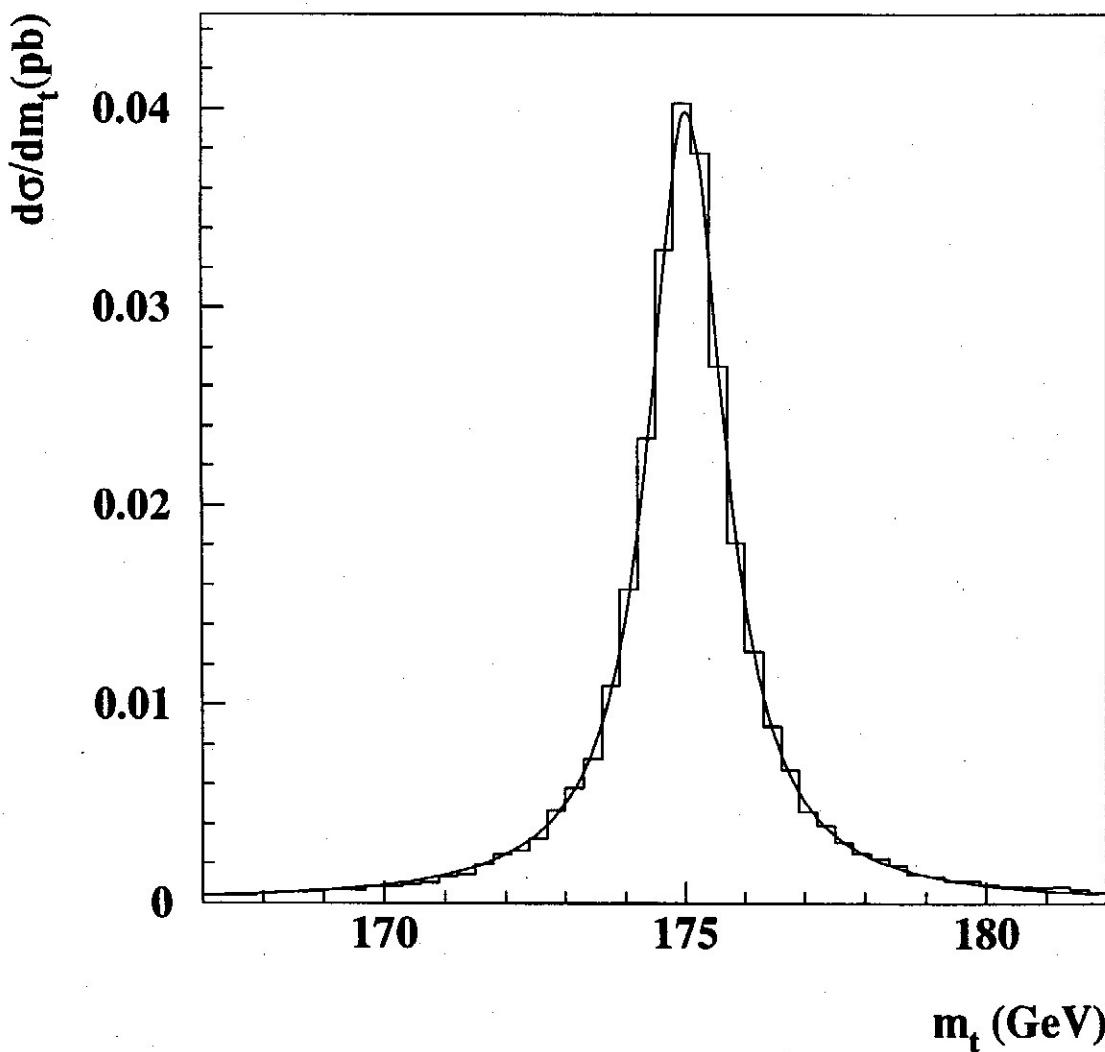
Results for $e^+e^- \rightarrow bW^+\bar{b}W^-g$

- three channel Monte Carlo, with
 - exact computation of matrix elements (using spinor techniques Stirling & Kleiss)
 - all spin correlations preserved
 - interference
 - exact kinematics
 - non-zero b mass
- studies of
 - top mass reconstruction
 - gluon radiation properties
 - interference effects

Mass Reconstruction

$E_{cm} = 500 \text{ GeV}$

$E_g > 10 \text{ GeV}$

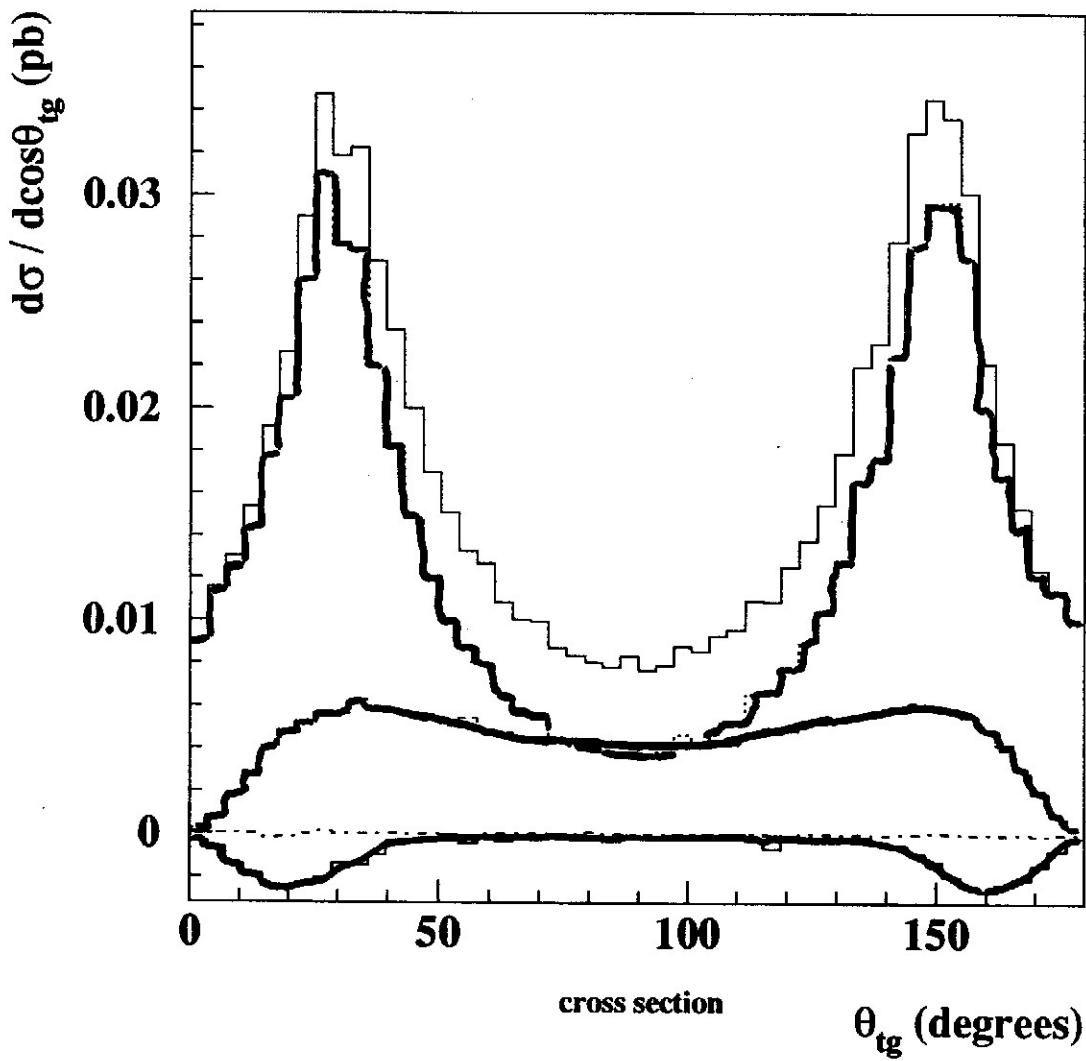


► used bg angle cuts

► Breit Wigner fit parameters

$$\begin{aligned} m &= 175.02 \text{ GeV} \\ \Gamma &= 1.53 \text{ GeV} \end{aligned}$$

$E_{cm} = 750 \text{ GeV}$
 $5 < E_g < 10 \text{ GeV}$
 $\cos\theta_{tb}, \cos\theta_{\bar{t}\bar{b}} < 0.9$



▷ exact Monte Carlo computation

- decay
- production
- decay-production interference
- decay-decay interference

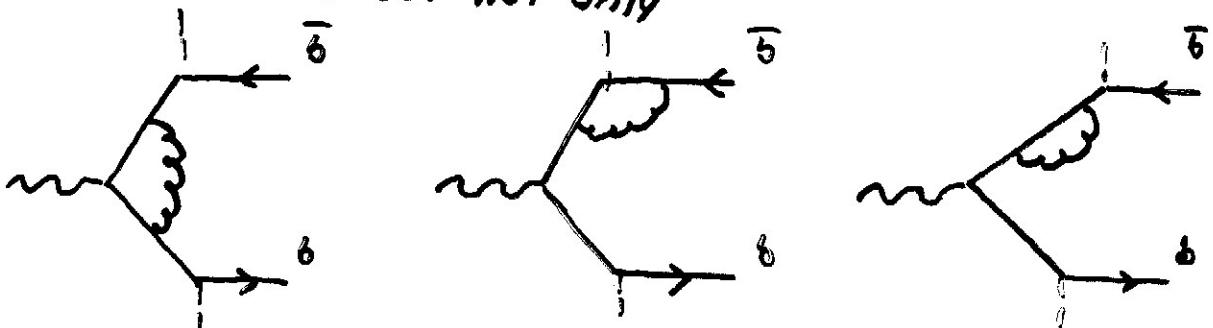


VIRTUAL CORRECTIONS

(to the doubly resonant diagram)

a) to subprocesses (factorizable-type corrections)

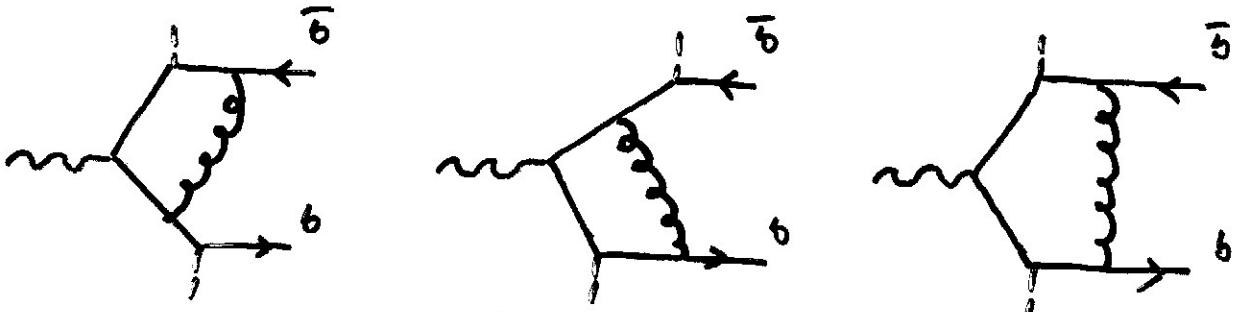
but not only



$$M \sim \frac{1}{p_t^2 - \bar{m}_t^2} \frac{1}{p_t^2 - \bar{m}_t^2}$$

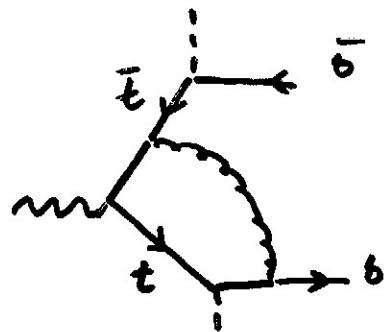
$$+ \frac{1}{p_t^2 - \bar{m}_t^2} \log(p_t^2 - \bar{m}_t^2) + \log(p_t^2 - \bar{m}_t^2) \frac{1}{p_t^2 - \bar{m}_t^2}$$

b) interference (*non-factorizable*)



$$M \sim \frac{1}{p_t^2 - \bar{m}_t^2} \log(p_t^2 - \bar{m}_t^2)$$

Amplitudes in DPA



Production-Decay Interference

- doubly resonant terms are entirely due to soft gluons
- employ *(extended) soft gluon approximation* (ESGA)

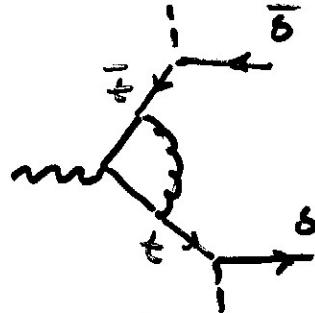
Denner, Dittmaier & Roth

⇒ contribution proportional to the tree level amplitude M_0 :

$$M_{b\bar{t}} = f_{b\bar{t}} M_0$$

(similar to results obtained in the W pair production case)

Vertex Corrections

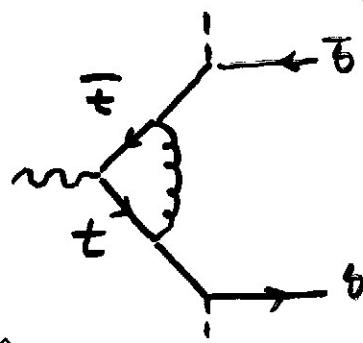


- production vertex:

$$M_{t\bar{t}} = M_{t\bar{t}}^{os} + M_{t\bar{t}}^{interf}$$

besides terms proportional to M_0 , $M_{t\bar{t}}^{interf}$ contains extra terms (unlike the W pair production case).

$$M_{t\bar{t}}^{interf} = f_{t\bar{t}} M_0 + M_1$$



3.2 Vertex Correction

$$M_{t\bar{t}} = \bar{u}(b) \hat{\epsilon}_{W^+} \frac{\hat{p}_t + m_t}{p_t^2 - \bar{m}_t^2} \delta\Gamma^\mu \frac{-\hat{p}_{\bar{t}} + m_t}{p_{\bar{t}}^2 - \bar{m}_t^2} \hat{\epsilon}_{W^-} v(\bar{b})$$

The vertex correction can be written :

$$\delta\Gamma^\mu = \frac{\alpha_s}{4\pi} *$$

$$[\begin{array}{lll} p_t^\mu F_1 & + & \gamma^\mu F_2 \\ + (\hat{p}_t - m_t) p_t^\mu F_3 & + & (\hat{p}_t - m_t) \gamma^\mu F_4 \\ + p_t^\mu (\hat{p}_{\bar{t}} + m_t) F_5 & + & \gamma^\mu (\hat{p}_{\bar{t}} + m_t) F_6 \\ + (\hat{p}_t - m_t) p_t^\mu (\hat{p}_{\bar{t}} + m_t) F_7 & + & (\hat{p}_t - m_t) \gamma^\mu (\hat{p}_{\bar{t}} + m_t) F_8 \end{array}]$$

- 8 form factors in the general case; on-shell, only F_1 and F_2 contribute
- $F_i \sim f_i C_{t\bar{t}}^0 + \dots \quad i = 1, \dots, 8,$

$$C_{t\bar{t}}^0 \sim \log(a(p_t^2, p_{\bar{t}}^2))$$

where $a(p_t^2, p_{\bar{t}}^2) \rightarrow 0$ when $p_t^2, p_{\bar{t}}^2 \rightarrow \infty$

\Rightarrow in DPA, keep F_1, \dots, F_6

- beside terms similar to on-shell computation (F_1, F_2), we acquire extra terms (F_3, \dots, F_6). This is due to the fact that our off-shell particle is a fermion.

Total cross-section

- Virtual corrections in DPA :

$$M^v = M_{t\bar{t}} + M_{tb} + M_{\bar{t}\bar{b}} + M_0(f_{b\bar{t}} + f_{\bar{t}b} + f_{b\bar{b}})$$

split into corrections to subprocesses + interference :

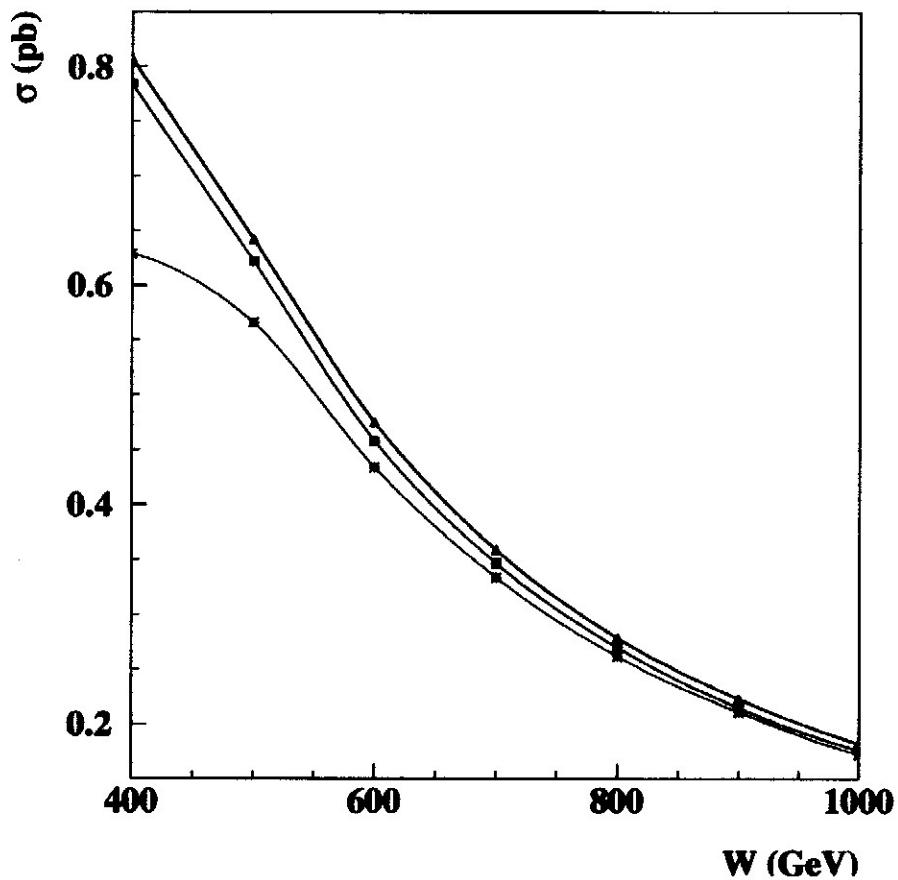
$$M^v = M_{prod}^v + M_{tdec}^v + M_{\bar{t}dec}^v + M_{interf}^v$$

$$M_{interf}^v = \alpha M_0 + \beta \log(p_t^2 - \bar{m}_t^2) M_1$$

- Infrared gluon divergencies
 - radiation from the on-shell particles (final state b 's) gives rise to IR singularities
 - radiation from the off-shell particles ($t\bar{t}$) is infrared safe
 - analytically integrate infrared gluons up to a cut-off energy of order 0.1 GeV (much smaller than the top width).

$$\begin{aligned} \sigma^{0+v} \sim & |M_0|^2 + (2M_0 \operatorname{Re}[M_{prod}^v]^* + |M_0|^2 I_{prod}) \\ & + \dots \text{decay} \quad \left. \right|_{\text{main}} \\ & + 2M_0 \operatorname{Re}[M_{prod}^v]_{\text{interf}}^* \longrightarrow \text{non-factorizable} \end{aligned}$$

- Add the real gluon radiation with $E_g > 0.1$ GeV



Parameters: $\alpha_s = 0.1$, $M_t = 175$ GeV,
 $\Gamma_0 = 1.55$ GeV, $\Gamma_1 = 1.42$ GeV

- *— tree level result, σ^0
- ▲— narrow width approx. result (C. Schmidt) $\equiv \sigma_{prod}^{0+1}$

$$d\sigma_{tot}^{0+1} = d\sigma_{prod}^0 \frac{d\Gamma_0^t \Gamma_0^t}{\Gamma_0^2} \left(1 - \frac{2\Gamma_1}{\Gamma_0} \right) + d\sigma_{prod}^1 \frac{d\Gamma_0^t \Gamma_0^t}{\Gamma_0^2} + \dots$$
- off shell top result, with cuts on the top invariant mass:

$$|\sqrt{p_t^2}, \sqrt{p_t^2 - M_t^2}| \lesssim 10 \Gamma_t \simeq 15 \text{ GeV}$$

Close to threshold, first order result is not reliable

$$\frac{\sigma^{NLO}}{\sigma^0} \simeq 1 + \alpha_s \frac{2\pi}{3\beta}, \quad \beta \rightarrow 0$$

Results for NLO corrections at W = 500 GeV

- narrow width approximation

-	σ_{prod}^{0+1}	-	using NLO top width	-	cuts on $m_t, m_{\bar{t}}$
	0.642 pb		0.660(2) pb		0.627(2) pb
-	interference : zero				

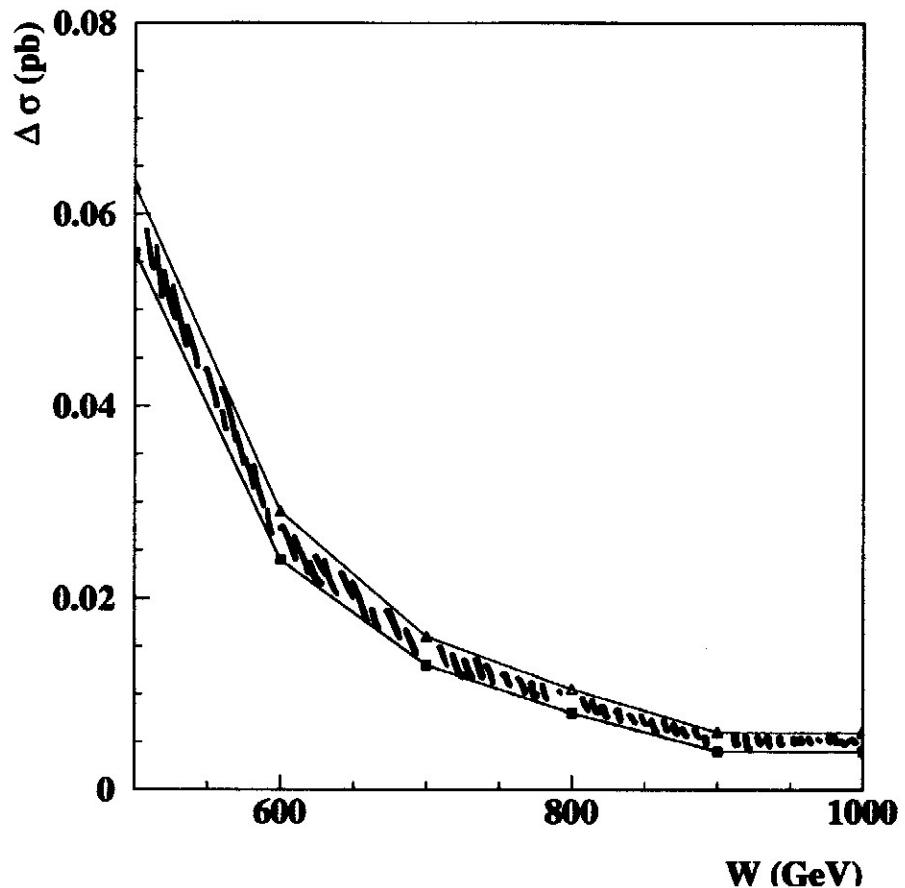
- off-shell top

- corrections to production and decay

-	-	no cuts	-	cuts on $m_t, m_{\bar{t}}$
		0.664(2) pb		0.629(2) pb

- interference

-0.0116(3) pb	-0.0068(2) pb
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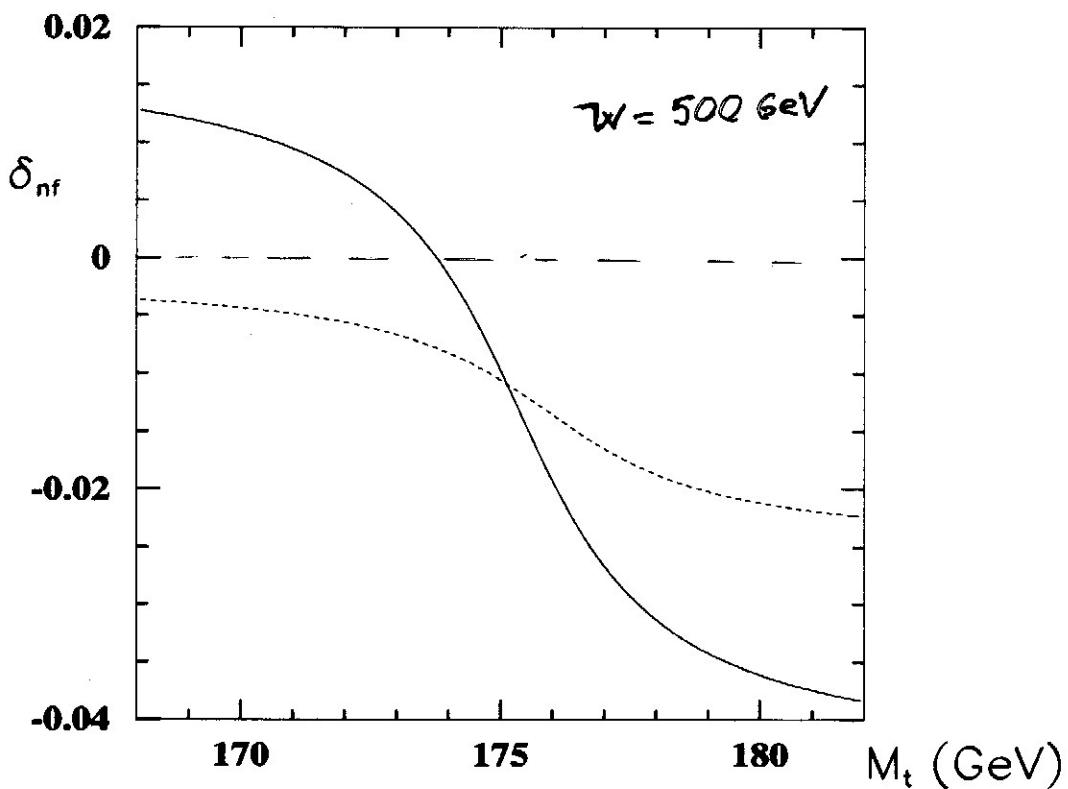
Off shell top

—▲— $\Delta\sigma = \sigma^{\text{main}} - \sigma^{\circ}$

—■— $\Delta\sigma = \sigma^{\text{tot}} - \sigma^{\circ}$

||||||| interference effects = $\sigma^{\text{tot}} - \sigma^{\text{main}}$

Relative magnitude of non-factorizable corrections *

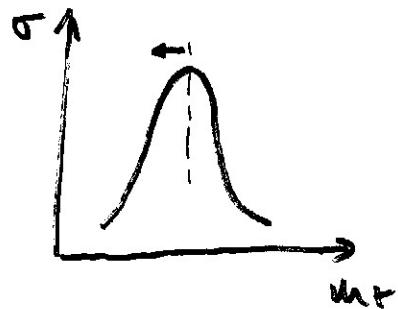


$$\delta_{nf} = \frac{\sigma_{nf}}{\sigma_0}$$

Off-shell computation, $|\sqrt{p_t^2} - 175| < 15$ GeV

- terms proportional to σ_0 only
- ... all terms

* can affect top mass reconstruction



On-shell DPA

- Alternative way of computing the interference effects

Corrections to production and decay

use on-shell approximation

Interference

analytic treatment of real gluon interference
only semi-soft gluons ($E_g \sim \Gamma$) contribute, use ESGA

- **Theorem** (Fadin, Khoze & Martin) :

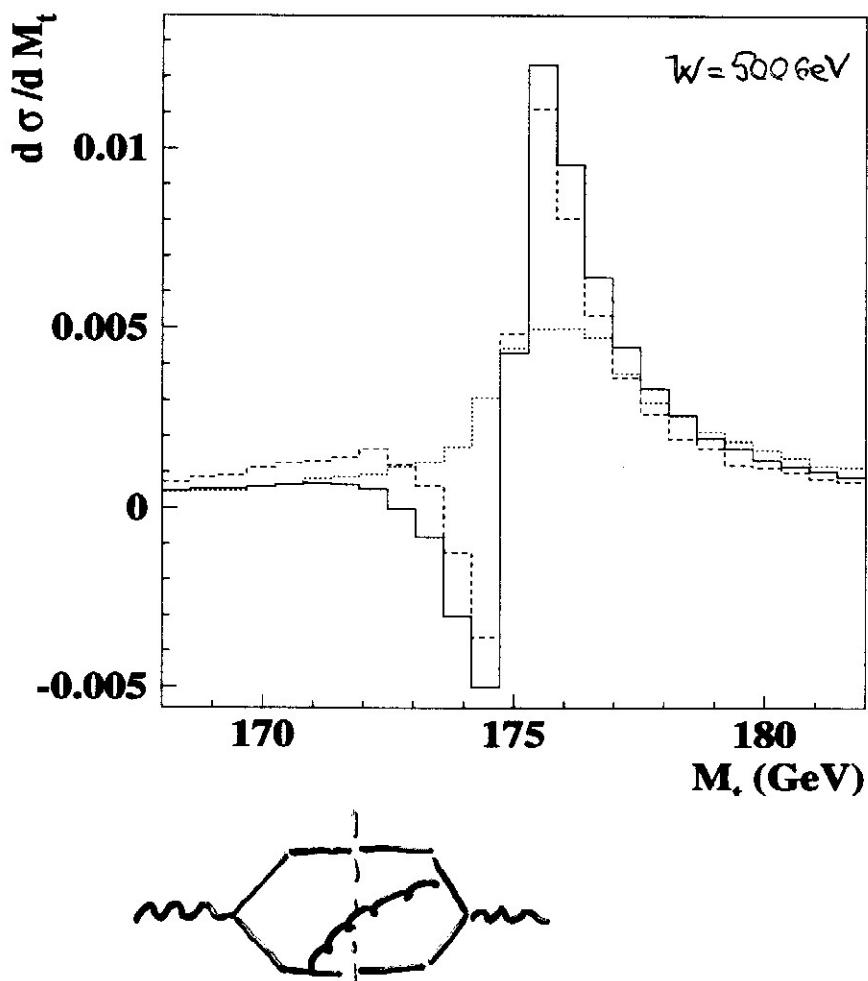
Interference effects are suppressed in inclusive quantities
are they zero ?

In this semianalytic approach (Denner, Dittmaier & Roth,
Beenakker, Berends & Chapovsky) virtual interference effects cancel the
real ones exactly.

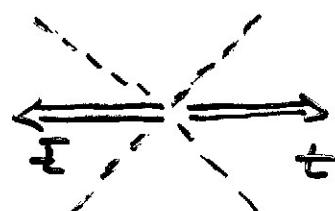
However:

- • this cancellation is dependent on the integration over all the invariant top mass range
- • there are ambiguities in the factorization of the phase space;
different choices lead to results differing by doubly resonant terms
(Denner, Dittmaier & Roth)

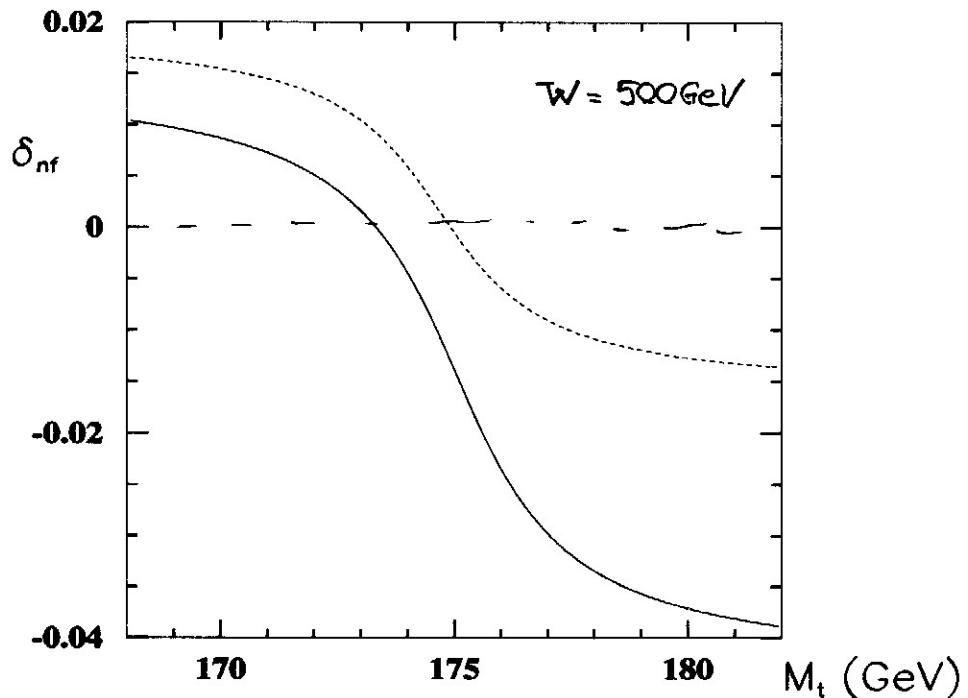
Comparision of results for real gluon radiation



- semianalytical computation, $\sigma = 0.121 \text{ pb}$
- - - exact (numerical) evaluation, $p_t = p_W + p_b$
- ... exact (numerical) evaluation, p_t reconstructed using gluon radiation angle, $\sigma = 0.124 \text{ pb}$



Relative magnitude of non-factorizable corrections



$$\delta_{nf} = \frac{\sigma_{nf}}{\sigma_0}$$

- ... on-shell computation (Beenakker, Berends & Chapuisky)
- off-shell computation, terms proportional to σ_0 only

Conclusions

- the top quark will play an important role in physics studies at the NLC
- need for improved precision of theoretical predictions above threshold
 - ◊ we have performed a DPA computation of QCD radiative corrections
 - exact (numerical) treatment of real gluon radiation
 - fully off-shell phase space and amplitude evaluation
 - ◊ comparision with results obtained through alternative methods :
 - terms which are **not** proportional to the Born amplitude contribute to interference
 - non-factorizable contributions are of order 1%
 - ◊ Plans for the future
 - add electroweak corrections to subprocesses
 - include non-resonant diagrams (at least at tree level)
 - SUSY corrections
- *full QCD corrections !?*